Melbourne School of Engineering Seminar

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von Kármán's Constant from Newton's Second Law.

The so-called von Kármán constant, κ , is the leading coefficient to the logarithmic equation for the profile of mean velocity, $U(y)$, for wall-bounded turbulent flows,

$$U^{+} = \frac{1}{\kappa} ln y^{+} + B, \tag{1}$$

where the superscript "+" in (1) denotes normalization by the dimensionally appropriate combination of kinematic viscosity and wall shear stress. The rather fundamental physical significance of κ derives from its obvious connection to how the turbulence near a wall, on average, distributes momentum. Many of the analyses associated with (1) either implicitly or explicitly, view κ as a constant in much the same way as other physical constants are viewed, e.g., Planck's constant. While (1) has been heuristically deduced by a number of means, its direct connection to the fundamental statement of dynamics has remained a long-standing challenge, as has the determination of κ from means other than empirically measuring the profile slope.

In this talk the results from a recently developed first-principles based theory are tested relative to the properties of (1). The theory demonstrates that that time averaged statement of Newton's second law formally admits a hierarchy of scaling layers, with an associated length scale distribution that asymptotically scales with distance from the wall. The results from direct numerical simulations (DNS) of the Navier-Stokes equations are shown to support these and other analytical findings. Specifically, the mean velocity profile exhibits logarithmic dependence (exact or approximate) when the solution to the mean momentum equation exhibits (exact or approximate) self-similarity on the hierarchy. Exact self-similarity corresponds to a constant leading coefficient in the logarithmic mean velocity equation. An independent equation for this coefficient (von Kármán's "constant") is derived. This equation and its various equivalent forms are shown to be in agreement with DNS data. Physically, κ exists owing to approximately scale-invariant dynamics over an internal layer hierarchy. The theory clarifies how and why logarithmic dependence occurs and that both logarithmic dependence and the constancy of κ are inherently approximate.