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Decomposition of turbulent fields using wavelets: an *a priori* investigation

P. Subbareddy[1], I. Marusic[1]

[1]Department of Aerospace Engineering and Mechanics, University of Minnesota
Minneapolis, MN 55455, USA.

Contact address: *marusic@aem.umn.edu, pramod@aem.umn.edu*

1 Introduction

The coefficients in a wavelet expansion are functions of both space and scale, enabling one to compute mixed space-scale statistics, which in the case of turbulent flow fields includes terms like energy transfers and other quantities [1], [2]. Of late, several research groups have proposed the use of wavelet expansions as efficient tools for the compression of the number of modes in turbulent flows. The essence of the idea is that the *coherent* part of the flow has a very small number of degrees of freedom as compared to the *incoherent*, random part of the flow field [3]. The compression involves filtering out coefficients which are smaller than a fixed threshold. This non-linear filtering is conjectured to partition the field into a part consisting of organised/coherent motions which are out of statistical equilibrium and one which is composed of random motions in statistical equilibrium. Proposed numerical methods use this compression to explicitly compute the coherent part of the vorticity, say, and use statistical models for the incoherent (nearly Gaussian) part [4], [5]. In this paper we consider three sets of turbulent fields for a preliminary analysis: homogeneous isotropic DNS, DNS of a channel flow and experimental data from stereoscopic PIV of a flat plate boundary layer. In the results to be presented, we use a 2D or 3D multiresolution analysis (MRA) [6] with orthogonal Coiflet-12 wavelets. For the isotropic DNS we use a 3D MRA and for the channel flow and PIV data sets, we use 2D MRA on planes parallel to the wall. This is justified by the assumption that the boundary layer data are homogeneous in the streamwise-spanwise planes.

2 Results and Discussion

The MRA is applied to each component of the vorticity field, ω_i, and a suitably defined threshold (ϵ) splits the wavelet coefficients, $\hat{\omega}_i$, into those larger than ϵ, $\hat{\omega}_i^f$, and those smaller than ϵ, $\hat{\omega}_i^r$. These are then inverse transformed

to give the filtered (coherent) and residual (incoherent) fields, ω_i^f and ω_i^r, respectively. Finding the right threshold is a key problem and the simplest thing to do is to find the flatness of the residual for a range of ϵ (a Gaussian signal will have flatness of 3). However, this is expensive and a threshold with no adjustable parameters is desirable. The "hard" threshold proposed in [7], given by $\epsilon_d = \sigma\sqrt{2\log N}$ has been adopted by Farge and co-workers. This is a useful threshold provided one can estimate σ (σ^2 is the variance of the noise in the signal and N is the grid size). If the noise is Gaussian, ϵ_d can be proved to be optimal in the sense of minimizing the $L2$ error [7].
Figure 1 shows some diagnostics for the isotropic DNS data (which is a 256^3 homogeneous isotropic simulation). Since the enstrophy ($Z = \frac{1}{2}\int \omega.\omega \mathrm{dV}$) is equally distributed among the 3 components of vorticity, one could use $\frac{2Z}{3}$ as an estimate for the variance of the noise itself, as proposed in [4], to find ϵ_d. The lines in Figures (1a,1b) indicate this estimate. Figure 2 shows isosurfaces of enstrophy of the original and reconstructed fields after filtering at ϵ_d. These results are in agreement with the results obtained by Farge and co-workers [4].

For boundary layers, 3D MRA of the same kind did not show much promise (at

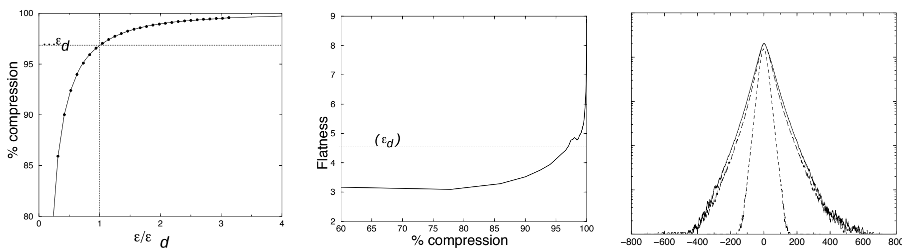

Figure 1: (a) Percent compression achieved as a function of the threshold applied. (b) Flatness of the residual field vs. percent compression achieved. (c) PDFs of the original, filtered(- -) and residual($\cdots$) vorticity field.

Figure 2: Isosurfaces of enstrophy of (a) the original unfiltered vorticity field and (b) Filtered (at $\epsilon = \epsilon_d$) vorticity field.

least with orthogonal wavelets and hard thresholding). This is due to the fact that the signal has considerable variation as one moves away from the wall (see Figure 3a). This prompted us to try 2D MRA on planes parallel to the wall. A range of thresholds were tried and some results for the channel flow and PIV data are given in Figure 3. The channel flow DNS is due to Kim, Moin and Moser [8] and has $Re_\tau \approx 590$. The PIV data has $Re_\tau \approx 1060$. As seen in the plots, using

ϵ_d (given by $\sigma_{\omega_i}\sqrt{2\log N}$ for the i component of the vorticity, ω_i) gives results similar to those from the isotropic data, at least in terms of compression rates and flatness factors. Figure 4 shows isosurfaces of enstrophy for the channel before and after filtering. The high degree of similarity is slightly deceptive, however, since the isosurfaces are evened out by the visualization program. The field is actually discontinuous in the wall-normal direction (as expected for 2D filtering on a 3D signal), and probably requires further processing (some kind of smoothing in the wall normal direction). Images of the PIV data before and after filtering are shown in Figure 5.

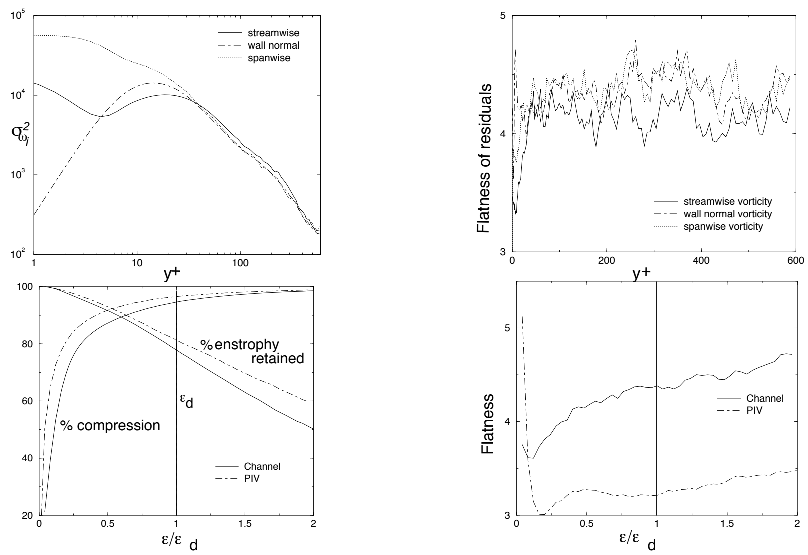

Figure 3: (a) RMS values of the vorticity vs. distance from the wall(wall units) for a turbulent channel flow DNS. This is used to compute the threshold (ϵ_d). (b) Flatness of the residual of the vorticity components at each wall normal plane after having been filtered at ϵ_d. (c) Compression and enstrophy retained for a particular plane ($y^+ \approx 200$) in the channel flow DNS and the PIV data. (d) Flatness as a function of threshold for the planes in (c).

3 Conclusions

In the boundary layer flows, the Donoho-Johnstone threshold when applied to individual wall normal planes, appears to function the same way it does with the isotropic flow, and seems to represent a good compromise between a large compression ratio and a flatness close to 3. However, it cannot be applied in the 3D case for these flows. Further work is required to treat wall bounded flows (or, in general, flows with large scale spatial/temporal variation). Additionally, some validation of the suitability of using a simple statistical model for the "incoherent

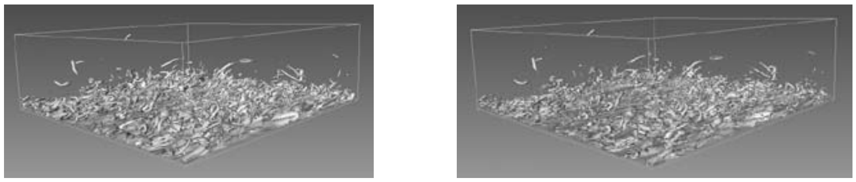

Figure 4: Isosurfaces of enstrophy in the channel of the (a) original unfiltered vorticity field and (b) filtered (at $\epsilon = \epsilon_d$) vorticity field.

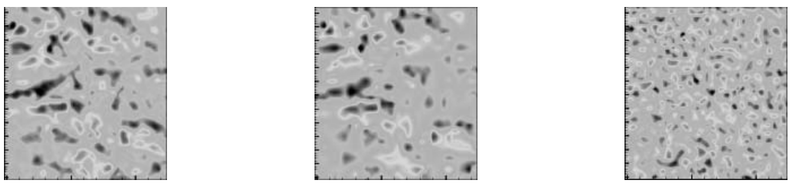

Figure 5: (a) Contour plot of wall normal vorticity, taken from the PIV of a turbulent boundary layer at $Re_\tau \approx 1060$. (b) After filtering at ϵ_d, (c) Residual field (Contour levels are smaller). (Views cover a field $\approx 1200 \times 1200$ wall units.)

part" is required.

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