

# Reynolds Number Dependence of the Amplitude Modulated Near-Wall Cycle

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**Abstract** The interaction in turbulent boundary layers between very large scale motions centred nominally in the log region (termed superstructures) and the small scale motions is investigated across the boundary layer. This analysis is performed using tools based on Hilbert transforms. The results, across a large Reynolds number range, show that in addition to the large-scale log region structures superimposing a footprint (or mean shift) on to the near-wall fluctuations, the small-scale structures are also subject to a high degree of amplitude modulation due to the large structures. The amplitude modulation effect is seen to become progressively stronger as the Reynolds number increases.

## 1 Introduction

Advances in numerical simulation, measurement techniques and high Reynolds number facilities have provided the opportunity in recent years to study in greater detail the relationship between eddying motions of different length scales in wall-bounded flows. The near-wall cycle, related to the near-wall streaks described by Kline *et al.* [11], has been largely viewed as depending only on global viscous wall units. The study by Jimenez & Pinelli [7] has shown that this region can self-sustain in the absence of an outer region and it is therefore often referred to as being autonomous. This autonomous view was based largely on an understanding of low Reynolds number flows, which by definition have a limited range of scales of motions. More recently, studies at higher Reynolds number (with high-fidelity measurement or simulation) have shown that the near-wall cycle is affected by the outer

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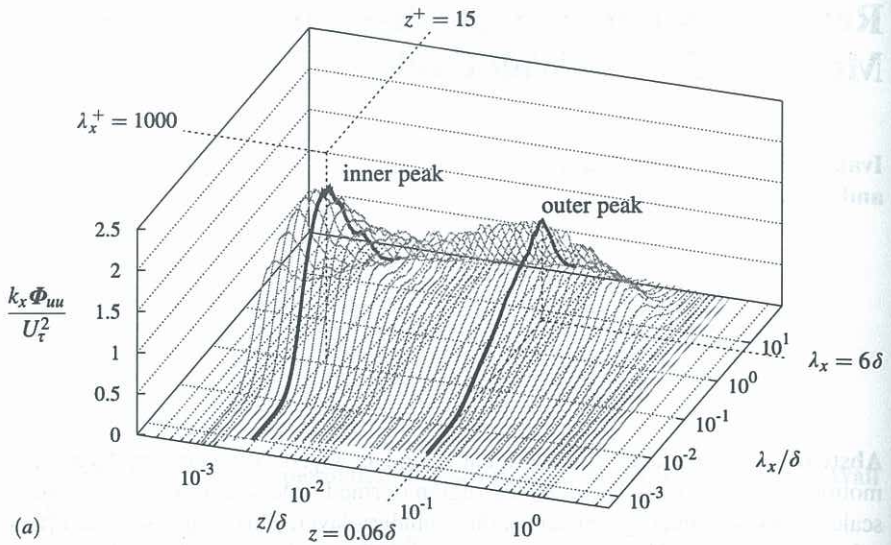


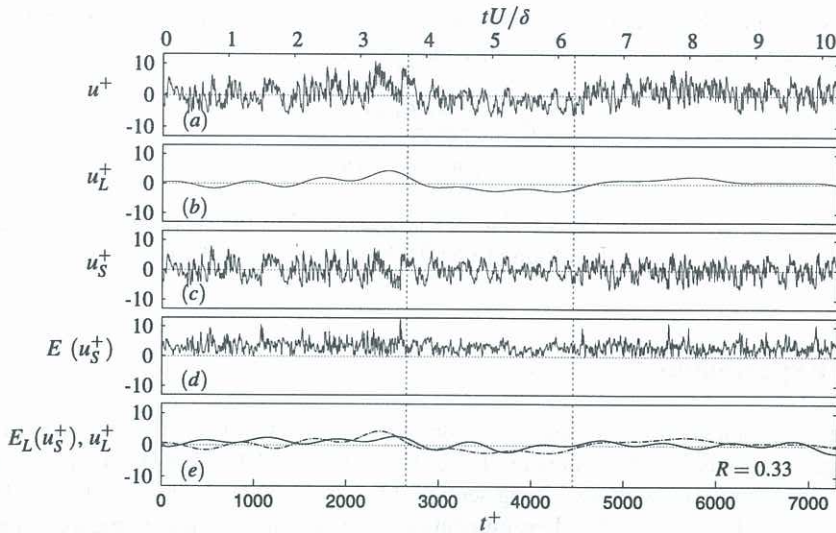
Fig. 1 Pre-multiplied energy spectrogram of streamwise velocity fluctuation  $k_x \phi_{uu} / U_\tau^2$  at  $Re_\tau = 7300$  across the turbulent boundary layer

flow region [1–3, 6, 8, 14, 16] and hence perhaps should not be considered as purely “autonomous”. Hutchins & Marusic [4] described the large-scale motions responsible for this as “superstructures”, with their origin nominally in the logarithmic region of the boundary layer. These observations came from experiments conducted at a range of Reynolds numbers, and Hutchins & Marusic [4] found that the strength (and influence) of the superstructures increased with increasing Reynolds number. Furthermore, they observed that low-wavenumber energy associated with these very large scale motions is not merely superimposed on the near-wall streamwise fluctuations, but seem to “amplitude modulate” the small-scale fluctuations [5].

This paper is concerned with this amplitude modulation interaction between the large and small (near-wall) scales in turbulent boundary layers. In the remainder of the paper the amplitude modulation effect will be quantified using a correlation coefficient and the effects of increasing Reynolds number will be considered. (A fuller discussion of these effects is given in a paper by Mathis *et al.* [15].)

## 2 Quantifying Amplitude Modulation

Hutchins & Marusic [4] showed that, at sufficiently high Reynolds numbers, two distinctive peaks appear in the pre-multiplied spectrogram of the fluctuating streamwise velocity, an example of which is shown in Fig. 1. Here the coordinate system,  $x$ ,  $y$  and  $z$ , refers to the streamwise, spanwise and wall-normal directions. The spectral density function of the streamwise velocity fluctuation is described by  $\phi_{uu}$



**Fig. 2** Example of small-scale decomposition on fluctuating  $u^+$  velocity signal at  $z^+ = 15$ ; (a) the raw signal  $u^+$ ; (b) the large-scale signal  $u_L^+$ ; (c) the small-scale signal  $u_S^+$ ; (d) its envelope; (e) filtered envelope (solid line) against the large-scale component (dot-dashed). For comparison, the mean of the filtered envelope has been adjusted to zero

and the streamwise wavenumber and wavelength are denoted by  $k_x$  and  $\lambda_x$  respectively (where  $\lambda_x = 2\pi/k_x$ ). The superscript “+” is used to denote viscous scaling ( $z^+ = zU_\tau/\nu$ ,  $u^+ = u/U_\tau$ , etc.).

The outer peak is related to superstructures. Hutchins & Marusic [4] showed that a very high level of correlation was found between the filtered (long wavelength) signatures of  $u$  simultaneously measured at the locations of the inner and outer peaks. This was understood to mean that the large-structures superimpose their “footprint” near the wall. To quantify the interaction we begin by decomposing the signal from a wall normal location corresponding to the inner peak site ( $z^+ = 15$ ).

Figure 2 shows a sample of the  $u$  signal at  $z^+ = 15$  for  $Re_\tau = 7300$ , as well as its large ( $u_L$ ) and small ( $u_S$ ) scale components. Here ‘large’ corresponds to a long-wavelength filtered signal ( $\lambda_x > \delta$  retained) and ‘small’ refers to the remainder ( $\lambda_x < \delta$ ). In order to determine the relationship between the large- and small-scale structures contained in any velocity signal, the small-scale component of the signal ( $u_S^+$ ) is analysed using the Hilbert transformation. This allows us to extract an envelope ( $E(u_S^+)$ ), representative of any modulating effect (assumed here to be the large-scale component  $u_L^+$ ). The envelope returned by the Hilbert transformation will track not only the large-scale modulation due to the log region events, but also the small-scale variation in the ‘carrier’ signal. To remove this effect, we filter the envelope at the same cut-off as the large-scale signal ( $\lambda_x/\delta > 1$ ). Hence, a filtered envelope ( $E_L(u_S^+)$ ) describing the modulation of small-scale structures is obtained.