The University of Melbourne
Department of Mechanical Engineering

Semester 1 Assessment 2009
436-431 Mechanics 4 Unit2 Dynamics

Reading time: 15 minutes **Writing time: two hours**

This paper has 4 Pages

Authorised materials

No materials are authorised

Calculators

No calculators allowed

Instructions to invigilators

Candidates should be issued initially with one 14-page script book

This paper may be taken by students at the end of the exam

Instructions to students

You are required to answer two questions.
Each question carries equal weight

Question 1.

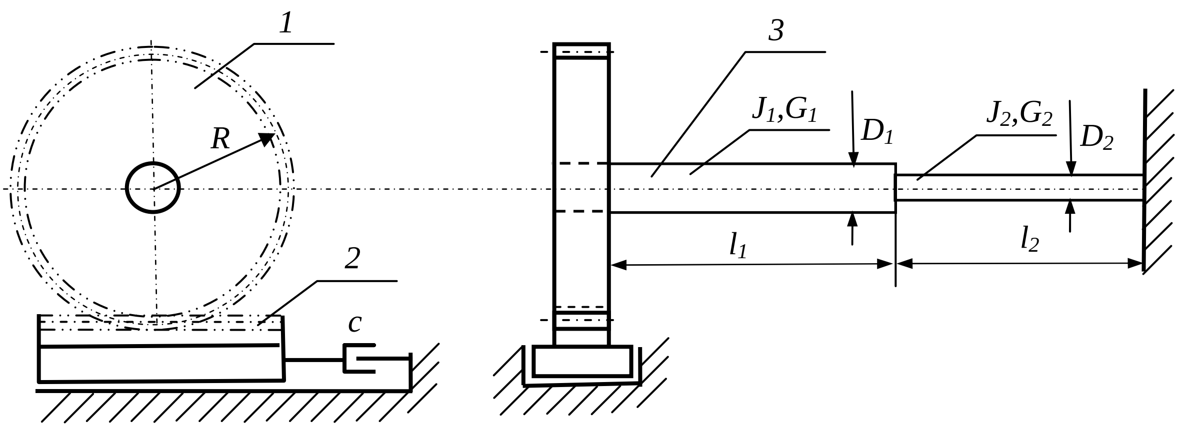

Fig. 1

In Fig. 1 the physical model for analysis of vibrations of a rack and pinion gear box is shown. The pinion *1* possesses the moment of inertia about its axis of rotation *I* and radius *R*. Rack *2* possesses mass *m*. The pinion is connected to the massless shaft *3*. Properties of the shaft are defined by the shear modulus *G* and the polar second moment of area *J*. The damper of the damping coefficient *c* models the energy dissipation.

Assuming that the system performs the underdamped ($\zeta<1$) vibrations, produce:

1. the differential equation of motion in the standard form
2. the expression for the period of the damped free vibrations
3. the expression for the damping ratio

At the instant of time equal to zero, the rack was displaced from its equilibrium position by distance x_0 and was released with the initial velocity equal to zero. Produce

4. the response of the system due to the above initial conditions
5. the time after which displacement of the system from its equilibrium position is less then $0.1x_o$

Question 2

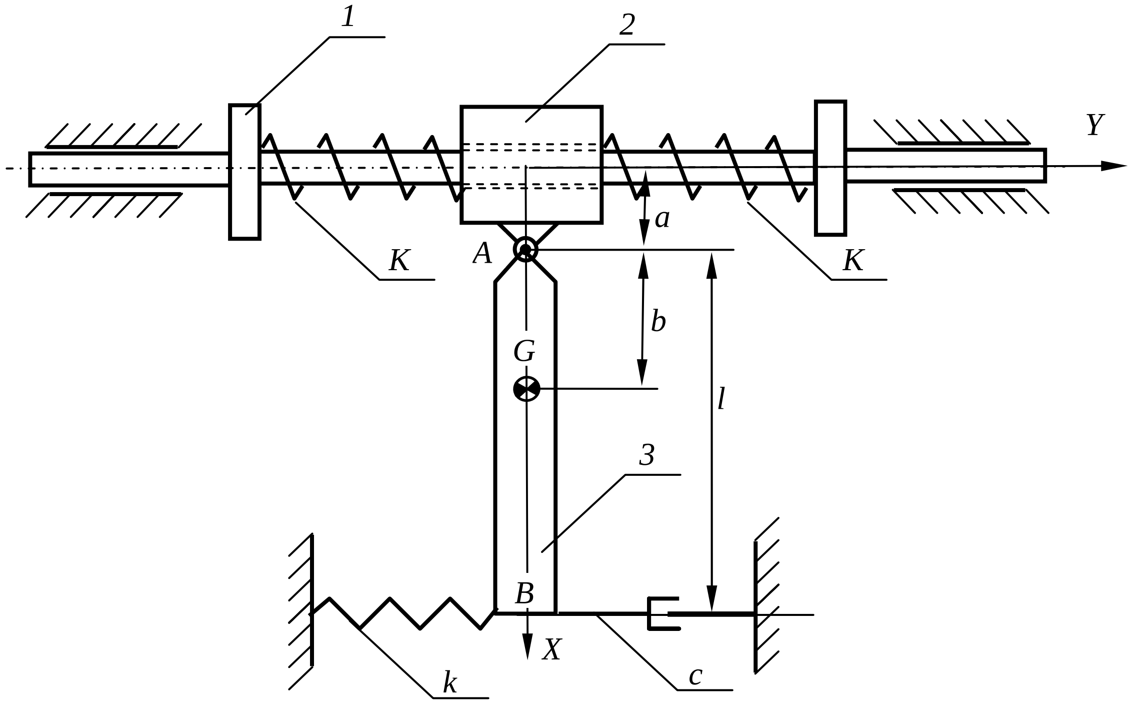

Fig. 2

In Fig. 2 the physical model of a mechanical system is shown. Slide *1* can move along the horizontal axis *Y* of the inertial system of co-ordinates *XYZ*. Its motion is governed by the following equation

$$Y = Y_0 \cos \omega t$$

Slider *2* possesses mass m_2 and is supported by two springs, each of stiffness *K*. At the point *A* body *3* is hinged to the slider *2*. Mass of the body *3* is equal to m_3 and its moment of inertia about the axis through its centre of gravity *G* is I_{G3}. At the point *B* the body *3* is supported by the spring of stiffness *k* and the damper of the damping coefficient *c*. Motion of the system takes place in the vertical plane *XY* and the system shown in the Fig 2 is at its equilibrium position.

Produce:

1. the differential equation of small oscillations of the system and present it in the following matrix form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(t)$$

(take advantage of the Lagrange's equations)

2. the expression for the amplitudes of the forced vibrations of the system due to the kinematic excitation Y(t)

3. the expression for the interaction force between body *2* and *3* at the point *A*.

Question 3

3.1. Prove the equality (1) for a system governed by the following equation

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^{\;2}x = 0$$

$$\ln\frac{x(t)}{x(t+T_d)} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \qquad (1)$$

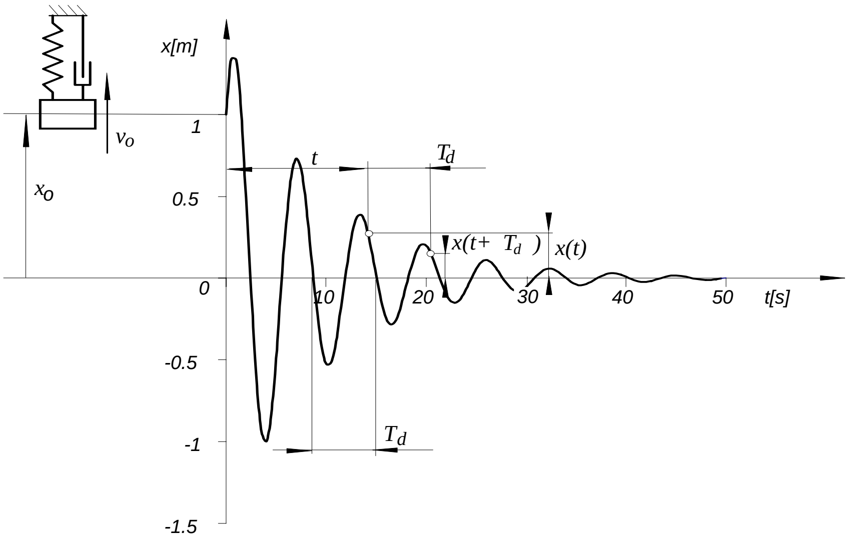

Fig. B3-2

3.2. Derive the expression for the amplitude of the forced vibrations of a system governed by the following equation

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^{\;2}x = q\sin\omega t$$

3.3. Produce the procedure for the balancing a rigid rotor in its own bearings.

END OF EXAMINATION PAPER