The University of Melbourne

Semester 1 Assessment, 2008

Department: MECHANICAL AND MANUFACTURING ENGINEERING

Subject Number: 436-353

Subject Title: MECHANICS 2

Exam Duration: THREE (3) HOURS

Reading Time: 15 minutes

This paper has **7** pages

Authorised Materials

No materials are autorised

Instructions to Invigilators:

Candidates should be issued initially with **two** 14-page script books.

Instructions to Students

This paper consists of two sections.
You are required to answer **two** questions from **each** section.
Answers to each of the sections will be completed in **separate answer booklets**.

Paper to be held in the Baillieu Library.

SECTION A – Stress Analysis

SECTION B – Mechanics of Rigid Bodies.

Answer only TWO questions in Section B. Each question carries equal weight.

Question B1

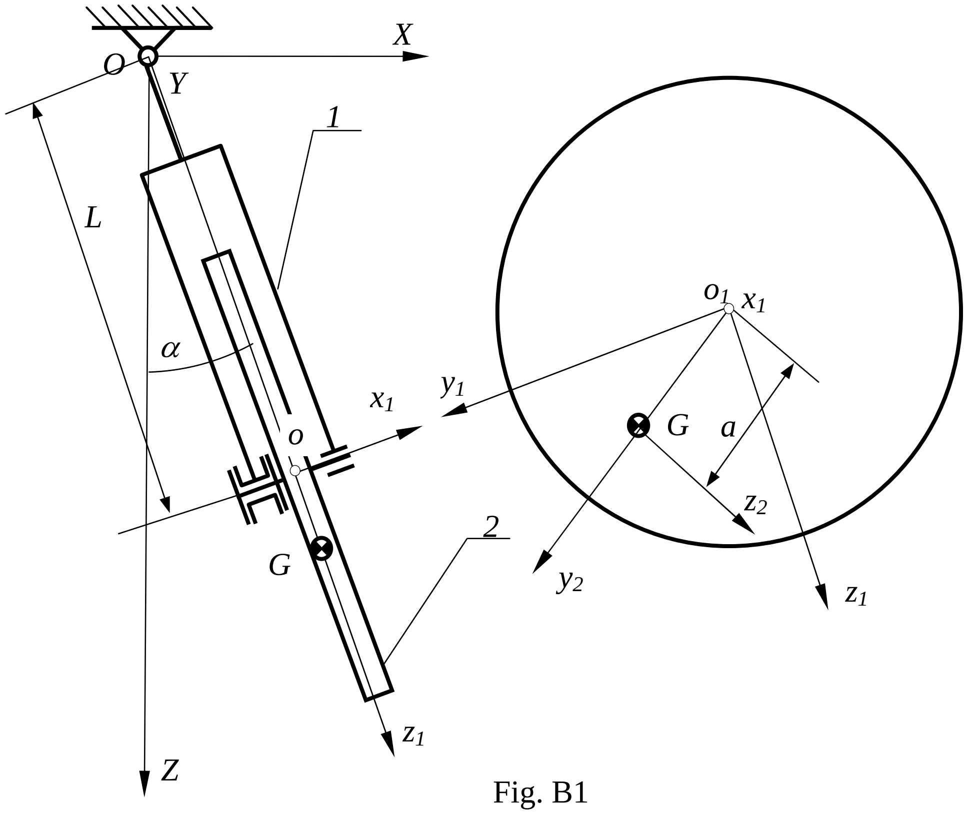

Fig. B1

The mount *1* of the wheel *2* oscillates about the horizontal axis *Y* of the inertial system of coordinates *XYZ*. Its motion is governed by the following function

$$\alpha = A \sin \omega t$$

The system of co-ordinates $x_1y_1z_1$ is rigidly attached to the mount. The relative angular velocity of the wheel *2* with respect to the mount *1* is constant and is equal to ω_{21}. The centre of gravity *G* of the wheel is off from the axis of relative rotation by the distance *a*. The system of co-ordinates $x_2y_2z_2$ is rigidly attached to the wheel. The wheel possesses mass *m* and its matrix of inertia about axes $x_2y_2z_2$ is

$$[I]_2 = \begin{bmatrix} I_{x2} & -I_{x2y2} & -I_{x2z2} \\ -I_{y2x2} & I_{y2} & -I_{y2z2} \\ -I_{z2x2} & -I_{z2y2} & I_{z2} \end{bmatrix}$$

Produce:

1. the expression for the components of the absolute angular velocity and acceleration of the wheel *2* along the $x_1y_1z_1$ system of co-ordinates
2. the expression for the components of the absolute liner velocity and acceleration of the point *G* along the $x_1y_1z_1$ system of co-ordinates
3. the expression for the kinetic energy of the wheel *2*

Question B2

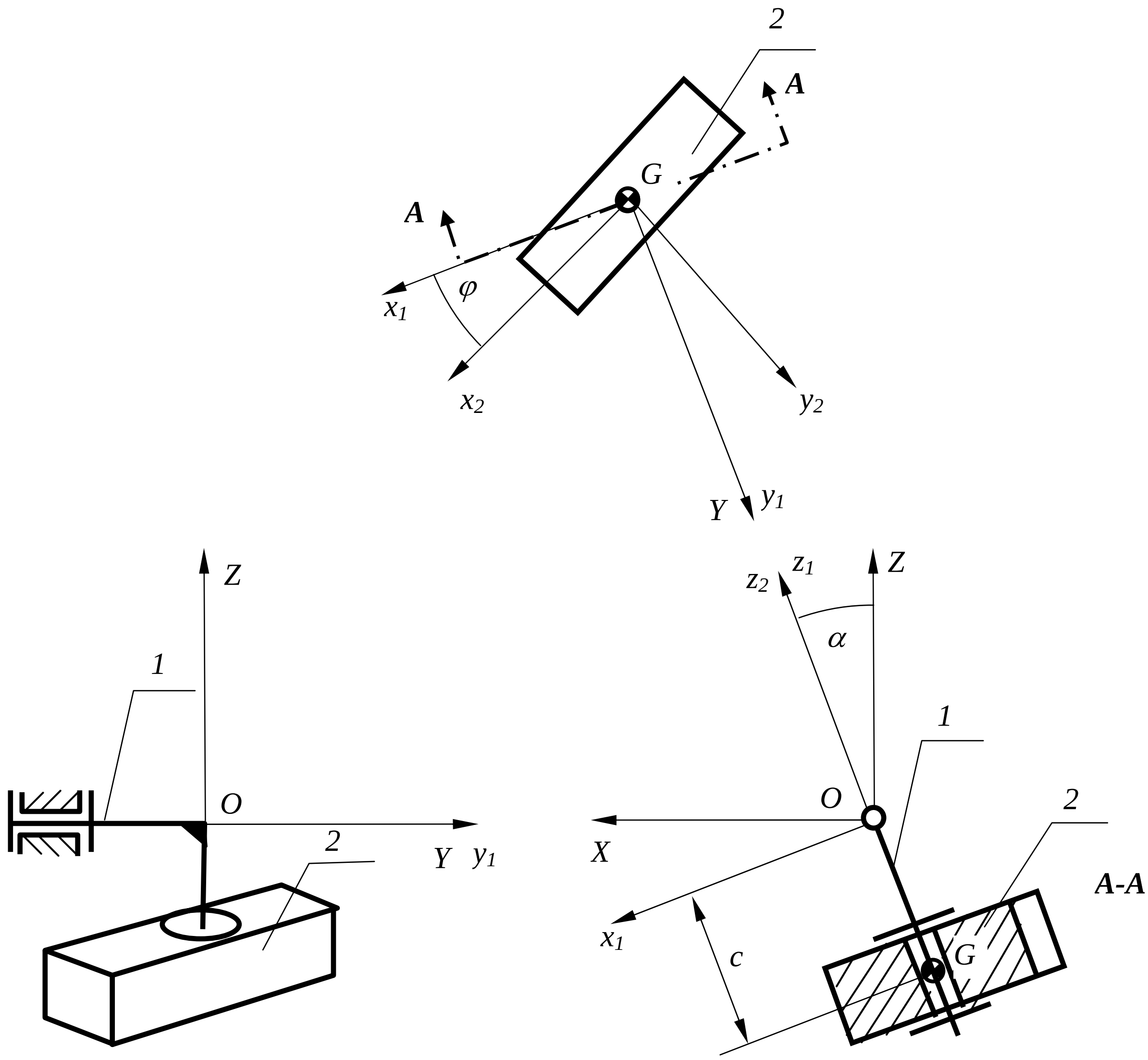

Fig.B2

The link *1* rotates about the horizontal axis *Y* of the inertial system of coordinates *XYZ*. The system of coordinates $x_1y_1z_1$ is rigidly attached to this link. Its angular position is defined by the angle α.

$$\alpha = A\sin\omega t$$

The rectangular block *2* is free to rotate about axis z_1. Its body system of coordinates $x_2y_2z_2$ goes through its centre of gravity *G* and coincides with the principal axes. The instantaneous position of this system of coordinates with respect to the system of coordinates $x_1y_1z_1$ is defined by the angle φ. The block possesses mass *m* and the principal moments of inertia about axes $x_2y_2z_2$ are I_{x2},I_{y2} and I_{z2} respectively.

The distance *c* locates the centre of gravity *G* of the block.

Produce:

1. the free body diagram of the body *1* (friction in all joints can be neglected)
2. the equations that allow you to produce both the relative motion of the block φ as a function of time and the interaction forces between the block *2* and the link *1*.

Question B3

B3-1. Prove that if $[I]_1$ is the matrix of inertia of a rigid body about the axes $x_1y_1z_1$, the matrix of inertia $[I]_2$ of this body about axis $x_2y_2z_2$ is given by the following formula

$$[I]_2 = [C_{1\to 2}]\,[I]_1[C_{1\to 2}]^T$$

where $[C_{1\to 2}]$ stands for the matrix of direction cosines between the system of coordinates $x_1y_1z_1$ and $x_2y_2z_2$.

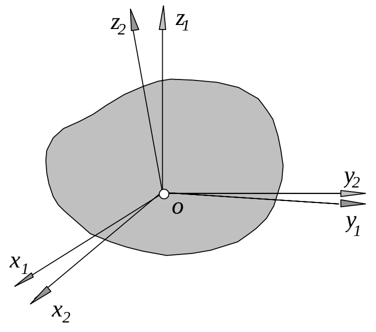

Fig. B3-1

B3-2. For a system of particles produce:
1. the definition of the linear momentum
2. the definition of the relative moment of momentum about a moving point

B3-3. Prove that rotation of the rigid body shown in Fig. B3-3 is governed by the following equations.

$$I_{Ox}\dot{\omega}_x + (I_{Oz} - I_{Oy})\omega_z\omega_y = M_{Ox}$$
$$I_{Oy}\dot{\omega}_y + (I_{Ox} - I_{Oz})\omega_x\omega_z = M_{Oy}$$
$$I_{Oz}\dot{\omega}_z + (I_{Oy} - I_{Ox})\omega_y\omega_x = M_{Oz}$$

where:
I_{Ox}, I_{Oy}, I_{Oz} - principal moments of inertia about axes through the point of rotation O
ω - absolute angular velocity of the body
$\mathbf{M}_O$ - resultant moment due to all external forces acting on the body about the point of rotation O

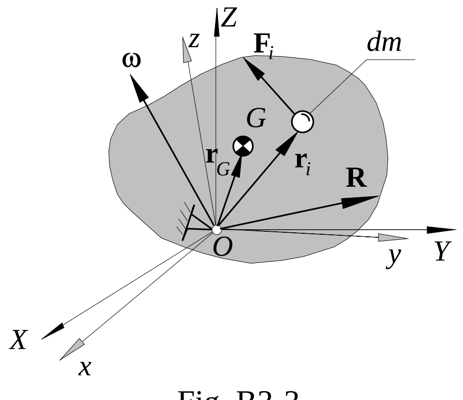

Fig. B3-3

End of Examination Paper